Errata to the paper "Consistency Techniques for Flow-Based Projection-Safe Global Cost Functions in Weighted Constraint Satisfaction"

J.H.M. Lee and K.L. Leung

In Section 4.2, Definition 15 and the proof for Lemma 1 should be refined as follows.

Definition 15 *A global cost function satisfies $\mathcal{FB}$ if:*

1. *W_S is flow-based, with the corresponding network $G = (V, E, w, c, d)$ with a fixed source $s \in V$ and a fixed destination $t \in V$;*
2. *there exists a surjective function M mapping each maximum flow f in G to each tuple $\ell \in \mathcal{L}(S)$ such that $min\{cost(f) \mid M(f) = \ell\} = W_S(\ell)$, and;*
3. *there exists an injection mapping from an assignment $\{x_i \mapsto v\}$ to a subset of edges $\bar{E} \subseteq E$ such that for all maximum flow f and the corresponding tuple ℓ_f, $\sum_{e \in \bar{E}} f_e = 1$ whenever $\ell_f[x_i] = v$, and $\sum_{e \in \bar{E}} f_e = 0$ whenever $\ell_f[x_i] \neq v$*

Lemma 1 *Given W_S satisfying $\mathcal{FB}$. Suppose W'_S is obtained from* `Project(`W_S`,`W_i`,`v`,`α`)` *or* `Extend(`W_S`,`W_i`,`v`,`α`)`*. Then W'_S also satisfies $\mathcal{FB}$.*

Proof: We only prove the part for projection, since the proof for extension is similar.

Assume $G = (V, E, w, c, d)$ is the corresponding flow network of W_S. We define $\bar{E}$ to be a set of edges corresponding to $\{x_i \mapsto v\}$. After `Project(`W_S`,`W_i`,`v`,`α`)`, G can be modified to $G' = (V, E, w', c, d)$, where $w'_e = w_e - \alpha$ if $e \in \bar{E}$, and $w'_e = w_e$ otherwise. We first show that W'_S satisfies conditions 1 and 2 with the corresponding network G'. Every maximum flow f in G can be applied to G'. We denote the flow cost of f in G and G' by $cost_G(f)$ and $cost_{G'}(f)$ respectively. We consider $W'_S(\ell)$ and $cost_{G'}(g)$ for every tuple $\ell \in \mathcal{L}(S)$, where $g \in \{f \mid M(f) = \ell\}$. If $\ell[x_i] = v$, $W'_S(\ell) = W_S(\ell) - \alpha$ and $cost_{G'}(g) = cost_G(g) - \alpha$, since $w'_e = w_e - \alpha$ whenever $e \in \bar{E}$. Otherwise, $W'_S(\ell) = W_S(\ell)$ and $cost_{G'}(g) = cost_G(g)$. Therefore, $W'_S(\ell) = min\{cost_{G'}(f) | M(f) = \ell\}$ holds for every tuple $\ell \in \mathcal{L}(S)$, *i.e.*

$$\begin{aligned} min\{W'_S(\ell) | \ell \in \mathcal{L}(S)\} &= min\{min\{cost_{G'}(f) \mid M(f) = \ell\} \mid \ell \in \mathcal{L}(S)\} \\ &= min\{cost_{G'}(f) \mid f \text{ is a maximum flow in } G'\} \end{aligned}$$

This proves condition 1. With similar arguments, condition 2 is also satisfied.

Moreover, since the topology of $G' = (V, E, w', c, d)$ is the same as that of $G = (V, E, w, c, d)$, W'_S also satisfies condition 3. ■

According to van Hoeve *et al.* (2006), all global cost functions listed in Section 5.1 satisfy the refined conditions as well and thus remain flow-based projection-safe.

Acknowledgments

We are grateful to Xuming Huang for discovering the inadequacy of Definition 15 in the original version of the paper and assisting in modifying the proof of Lemma 1.

References

van Hoeve, W., Pesant, G., & Rousseau, L.-M. (2006). On Global Warming: Flow-based Soft Global Constraints. *J. Heuristics*, *12*(4-5), 347–373.